Unit 4

Describing Functions

Functions can be represented using tables, graphs, equations, and words. In this unit, you will explore what makes a relationship a function and how functions can model situations and tell stories. You will use function notation to describe key features of functions, compare different functions, and define functions. Finally, you will explore new types of functions that can model situations that have different rules for different inputs.

Essential Questions

- What are the characteristics of a function and how can a function tell a story?
- What are key features of functions and how can you describe them?
- How can you use function notation as a tool to communicate precisely?

Summary | Lesson 1

We can represent rules with verbal descriptions or as a table of *inputs* and *outputs*. All sets of inputs and outputs are called *relations*. A *function* is a special kind of relation that assigns exactly one output to each possible input.

You can determine whether a rule is a function by organizing the inputs and outputs into a table. If one input has multiple possible outputs, then the rule is not a function.

Here are two examples.

Rule A takes an integer and outputs an integer that is one less.

Input	Output
1	0
2	1
2	1
4	3

In this relationship, Rule A is a function because each input has exactly one output.

Rule B takes a number and outputs a random number that is greater.

Input	Output
0	2
0	10
-2	0
-1.6	-1.2

In this relationship, Rule B is *not a* function because each input has multiple outputs.

Try This

This rule takes any value and either multiplies or divides it by 2.

Is this rule a function? Explain your thinking.

Input	Output
2	4
10	20
3	6
2	1

Summary | Lesson 2

Function notation is a way to write the inputs and outputs of a function. For example, $f(4) = 9$ is a statement written in function notation. It says that when the input of the function f is 4, the output is 9. In other words, when the value of the *independent variable* is 4, the value of the *dependent variable* is 9.

This table shows some input-output pairs for the function $s(t)$ that determine the price of a slice of pizza based on the number of toppings.

$s(2) = 2.75$ is a statement written in function notation.

- $s(2)$ can be read as "s of two."
- For this situation, the number of toppings is the *independent variable*, t, and the price of a slice of pizza is the *dependent variable*, $s(t)$.
- $s(2) = 2.75$ means the price of a slice of pizza with 2 toppings is \$2.75.

Menu

Slice of Pizza
\$1.75 plus \$0.50 per topping

Number of Toppings	Price (\$)
0	1.75
1	2.25
2	2.75

Try This

An ice cream shop serves ice cream in either a waffle cone or a bowl.

$w(x) = 2.25x + 3.5$ represents the cost of ordering a waffle cone, where x is the number of scoops of ice cream.

a What is the value of $w(2)$?

b What does $w(4) = 12.5$ mean in this situation?

Summary | Lesson 3

You can represent function rules with equations, verbal descriptions, and tables.

For example, the function $s(t)$ describes the relationship between the cost of a slice of pizza and the number of toppings, t.

Description

Menu
Slice of Pizza
$1.75 plus $0.50
per topping

Table

Number of Toppings	Price ($)
0	1.75
1	2.25
2	2.75

Equation

$s(t) = 1.75 + 0.50t$

You can use the equation to determine different values of the function.

Let's determine the value of $s(4)$:

$s(4) = 1.75 + 0.50(4)$

$s(4) = 3.75$

This means the price of a slice of pizza with 4 toppings is $3.75.

Try This

An ice cream shop serves ice cream in either a waffle cone or a bowl.

A bowl of ice cream costs $1.75 plus $2.25 for each scoop of ice cream.

a Write an equation for $b(x)$ where b represents the cost of a bowl of ice cream and x represents the number of scoops.

b What is the value of $b(3)$?

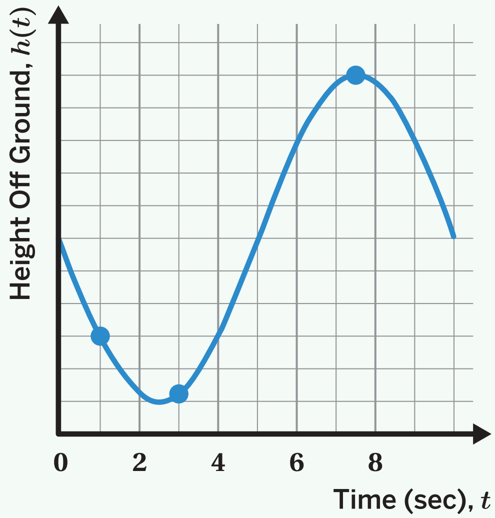
Summary | Lesson 4

A graph can reveal in more detail what is happening during a situation. Here is an example.

The function $h(t)$ represents the height of the cart on the Ferris wheel at time t.

We can use the graph to describe many parts of the situation. For example:

- At around 7.5 seconds, the Ferris wheel cart is at its maximum height.
- $h(1)$ is greater than $h(3)$. This means the Ferris wheel cart was higher off the ground at 1 second than at 3 seconds.

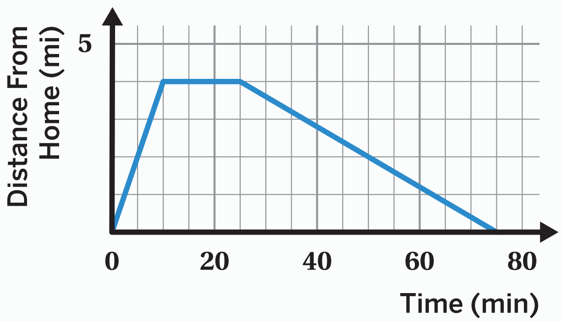


While we can use the graph to describe many things, there are lots of things the graph cannot describe. For example:

- How much fun the people are having
- How many people are riding the Ferris wheel

Try This

Jasmine takes the bus to the library and walks home. This graph shows Jasmine's distance from home $d(t)$.



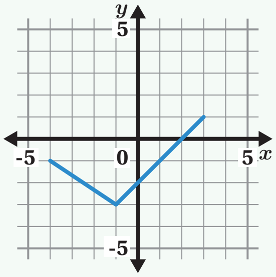
a What does $d(15) = d(20)$ represent in this situation?

b What does $d(50)$ represent in this situation?

Summary | Lesson 5

We can use the key features of a graph to help us describe a function or sketch a possible graph of a function. Here is an example. Let's analyze the graph of this function.

Minimum: The lowest point on a graph.	$(-1, -3)$
Maximum: The highest point on a graph.	$(3, 1)$
Positive: The x-values where the function has positive outputs; the graph is *above* the x-axis.	$x > 2$
Negative: The x-values where the function has negative outputs; the graph is *below* the x-axis.	$x < 2$
Increasing: The x-values where the graph is sloping upward as you read the graph from left to right. As the inputs increase, the outputs also increase.	$x > -1$
Decreasing: The x-values where the graph is sloping downward as you read the graph from left to right. As the inputs increase, the outputs decrease.	$x < -1$

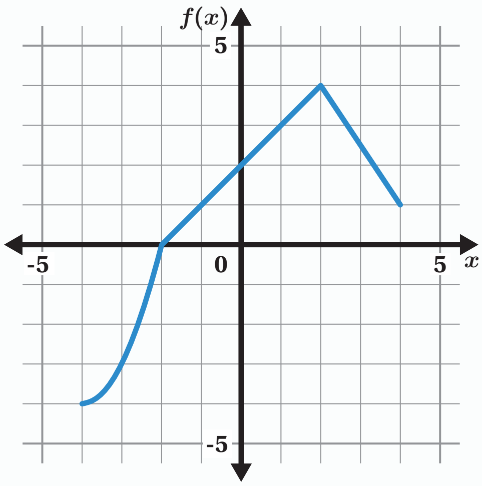


Try This

Select *all* the statements that are true for $f(x)$.

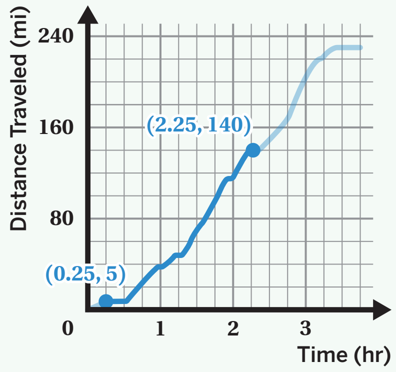
$f(x)$ . . .

- ☐ **A.** Is positive when $x > -2$.
- ☐ **B.** Is increasing when $x > -2$.
- ☐ **C.** Is decreasing when $x > 2$.
- ☐ **D.** Has a minimum at $(-4, -4)$.
- ☐ **E.** Has a maximum at $(0, 4)$.



Summary | Lesson 6

Functions can have different rates of change over different intervals. The **average rate of change** is equivalent to the *slope* of the line between two points.

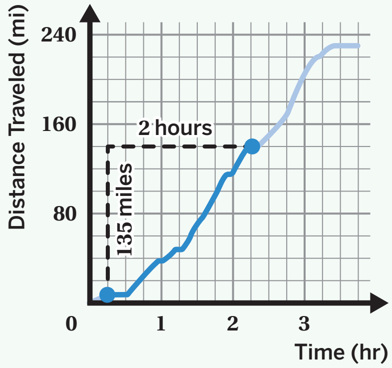


This graph represents Troy's car trip.

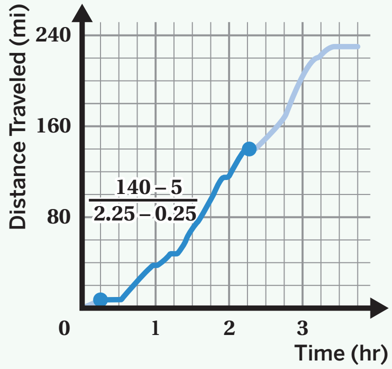
We can calculate the average rate of change over an **interval**, a specific length between two points, like the interval from 0.25 to 2.25 hours.

Here are two different strategies.

Strategy 1



Strategy 2



The average rate of change for the interval 0.25 to 2.25 hours is $\frac{135}{2} = 67.5$.

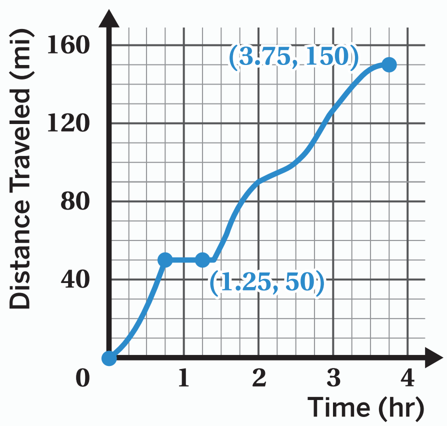
That means that Troy's average speed was 67.5 miles per hour in that interval.

Try This

Oscar took the train to attend his friend's birthday. Here is a graph of his trip.

Determine Oscar's average rate of change for the interval 1.25 to 3.75 hours.

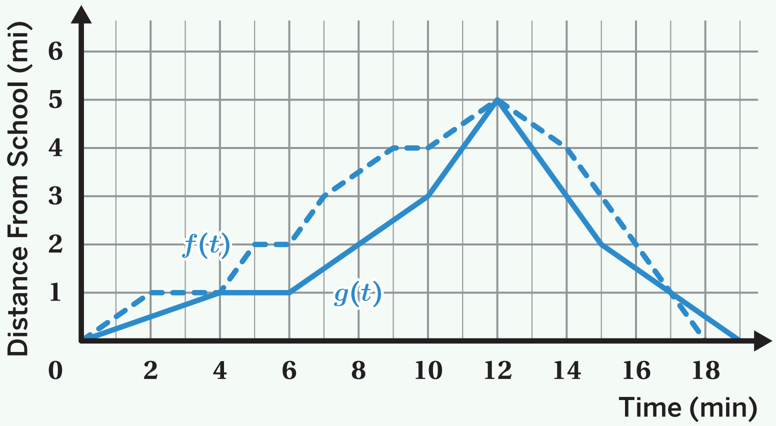
Show or explain your thinking.



Summary | Lesson 7

We can analyze functions by comparing their key features at different intervals and then use function notation to describe them.

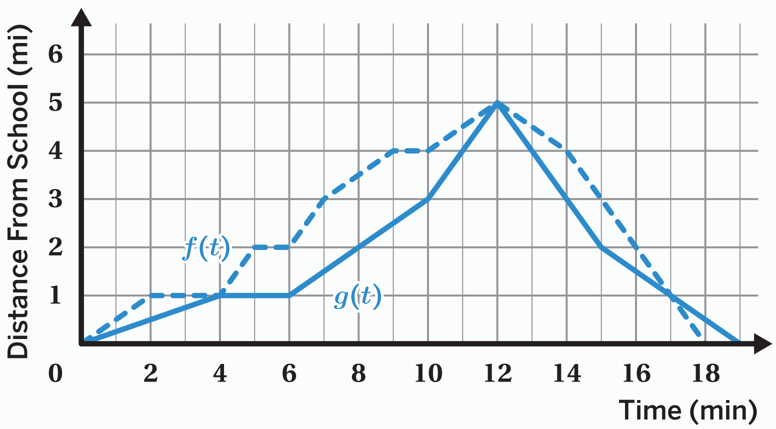
For instance, here are some true statements about these two graphs:



- When $t = 4$, $f(t) = g(t)$.
- $f(8) > g(8)$
- $f(12) = g(12)$
- $f(15) > g(15)$
- $f(t)$ and $g(t)$ have the same maximum.
- $f(t)$ and $g(t)$ are both decreasing from 12 to 15 minutes.
- $f(t)$ and $g(t)$ have the same average rate of change from 5 to 6 minutes.

Try This

A school has two buses that take different routes to drop students off. $f(t)$ and $g(t)$ represent the distance of each bus from school (in miles) after t minutes.



Select *all* the true statements:

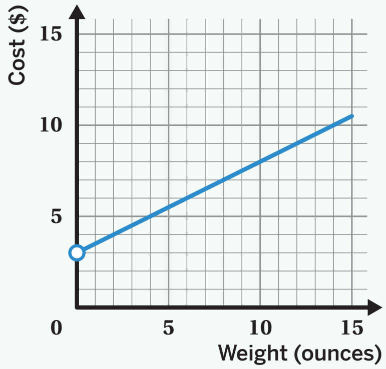
- ☐ **A.** $f(6) = g(6)$
- ☐ **B.** $f(10) > g(10)$
- ☐ **C.** $f(17) = g(17)$
- ☐ **D.** $g(5) = 1$
- ☐ **E.** $f(18) > g(18)$

Summary | Lesson 8

The **domain** of a function is the set of all possible input values (or all possible values for the *independent variable*). The **range** is the set of all possible output values (or all possible values for the *dependent variable*).

Having context for what the function is describing helps to make sense of possible inputs and outputs. Some domains and ranges are *discrete* while others are *continuous*. *Discrete* means that the possible values are distinct or separate, like the whole numbers from 1 to 10. *Continuous* means that all numbers in an interval are possible. In a graph, this means that there are no breaks, jumps, or holes.

You can find the domain by looking at the x-values as you read the graph from left to right. You can determine the range by looking at the y-values as you read the graph from bottom to top.



Here the function $f(w) = 3 + 0.5w$ represents the cost of a frozen yogurt that weighs w ounces.

To determine the domain, consider which inputs make sense for the situation. Someone could buy 0.5 or 2 ounces of frozen yogurt, but not 0 or -2 ounces, so the domain is all numbers greater than 0.

To determine the range, consider which outputs make sense. Someone could pay $\$4$ or $\$6.25$ for frozen yogurt, but not negative dollars. It also doesn't make sense for frozen yogurt to cost $\$3$ because that would mean the customer purchased 0 ounces of yogurt. This means that the range is all amounts greater than 3 up to the nearest cent.

Try This

A local mechanic sells and replaces tires. The total cost of replacing tires is $\$50$ for labor plus $\$100$ for each tire.

The function $r(t) = 50 + 100t$ represents the total cost for t tires.

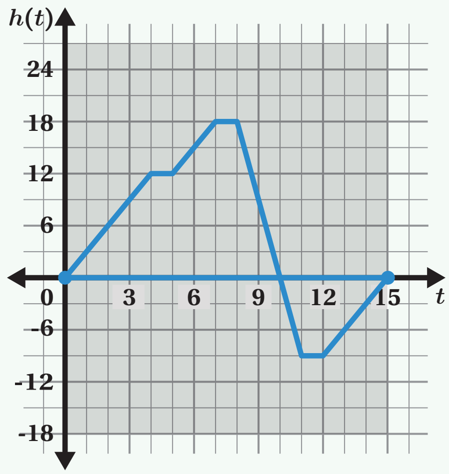
a Select *all* the values that are in the domain of $r(t)$.

- ☐ **A.** -2
- ☐ **B.** 0
- ☐ **C.** 1
- ☐ **D.** 4
- ☐ **E.** 1.5

b Describe the domain of $r(t)$.

Summary | Lesson 9

The *domain* and *range* of a function can each be described using a **compound inequality**, which is two or more inequalities joined together. You can write a compound inequality using symbols or using the words "and" or "or".



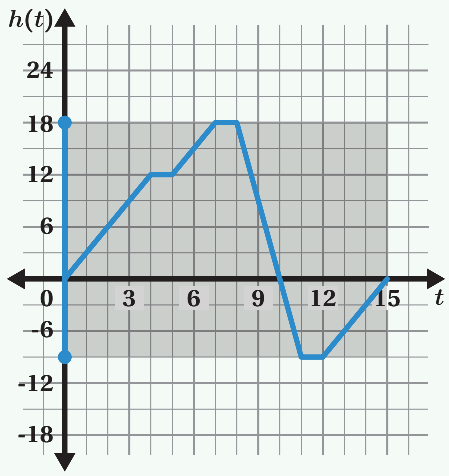
A graph can help you visualize the domain and range of a function, making it easier to describe them using compound inequalities.

Domain: The width of the function, or how far left and right the function goes. The domain is also the set of all inputs to the function, or all values of the independent variable. The domain of this function is all the t values from 0 to 15.

$t \geq 0$ and $t \leq 15$

$0 \leq t$ and $15 \geq t$

$0 \leq t \leq 15$



Range: The range describes the height of the function, or how far up and down the function goes. The range is also the set of all outputs of the function, or all values of the dependent variable. The range of this function is all the $h(t)$ values from -9 to 18.

$h(t) \geq -9$ and $h(t) \leq 18$

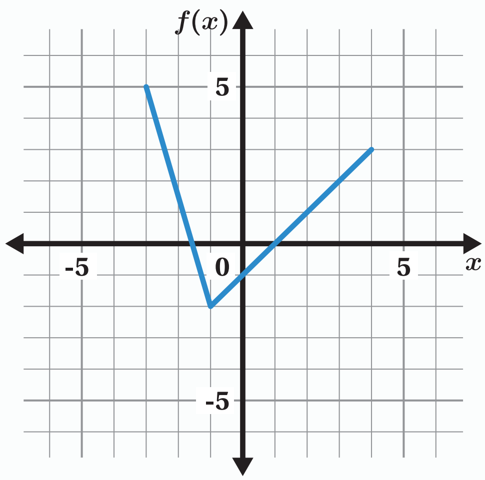
$-9 \leq h(t)$ and $h(t) \leq 18$

$-9 \leq h(t) \leq 18$

Try This

Match the domain and range of $f(x)$ with one of these compound inequalities:

$-5 \leq x \leq 4$	$-2 \leq f(x) \leq 5$
$-3 \leq f(x) \leq 7$	$-3 \leq x \leq 4$
$-3 \leq x \leq 5$	



Domain: ________________

Range: ________________

Summary | Lesson 10

You can restrict a function's domain or range to highlight specific portions of a graph. Inequalities are one way to represent these restrictions symbolically.

Here is an example. Let's restrict the domain and range of $h(x)$ to highlight the interval from $(-3, 7)$ to $(6, 1)$.

To restrict the domain, use the x-values of each ordered pair:

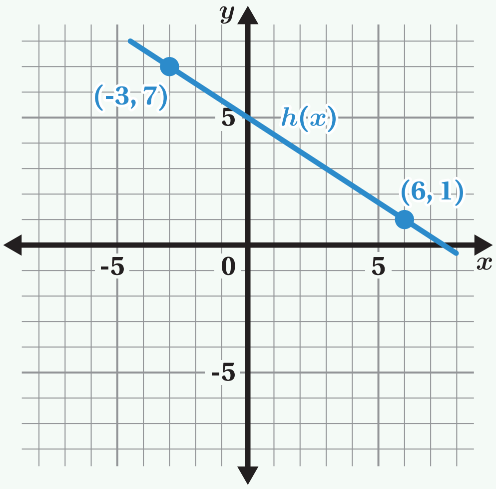
Domain: $-3 \le x \le 6$

When you restrict the domain, you are restricting the x-values, which should be included when you write the inequality.

To restrict the range, use the y-values of each ordered pair:

Range: $1 \le h(x) \le 7$

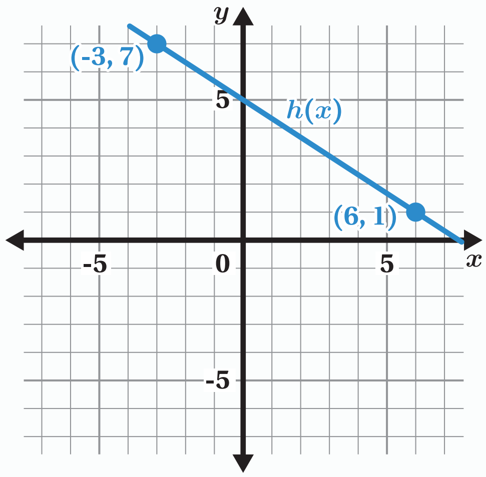
When you restrict the range, you are restricting the y-values, or the output values of $h(x)$.



Try This

a Write a domain that restricts the graph of $h(x)$ from $(-3, 7)$ to $(6, 1)$.

b Write a range that restricts the graph of $h(x)$ from $(-3, 7)$ to $(6, 1)$.



Summary | Lesson 11

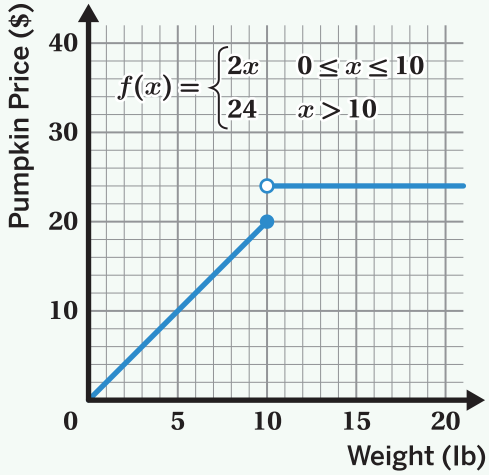
A **piecewise-defined function** is a function in which different rules apply to different intervals in its domain. You can use a graph or an equation to evaluate a piecewise-defined function.

Let's look at an example. The function $f(x)$ represents the price of a pumpkin with a weight of x pounds.

You can use the graph to evaluate $f(4)$ and $f(15)$.

- The point $(4, 8)$ is on the graph, so $f(4) = 8$.
- The point $(15, 24)$ is on the graph, so $f(15) = 24$.

You can also use the equation to evaluate $f(4)$ and $f(15)$.



Value	Domain Interval	Equation	Evaluate
$f(4)$	$x = 4$ is in $0 \le x \le 10$	$f(x) = 2x$	$f(4) = 2(4)$ $= 8$
$f(15)$	$x = 15$ is in $x > 10$	$f(x) = 24$	$f(15) = 24$

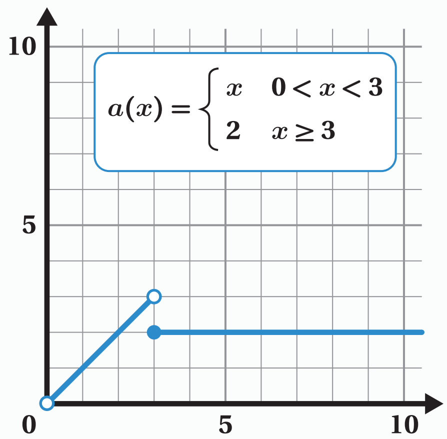
Try This

Here is a graph of a piecewise-defined function.

a Why is this a piecewise-defined function?

b What is $a(1)$?

c What is $a(13)$?



Summary | Lesson 12

You can use *piecewise-defined functions* to represent situations. A **step function** is one special kind of piecewise-defined function, where every section of the graph is a point or a horizontal line at a constant value.

Here are some helpful ways to write the equations of a piecewise-defined function:

- The number of conditions in an equation is equal to the number of pieces in the function and graph.
- In the piecewise equation, each piece represents one condition and has its own domain.
- You can write the domain as an inequality.
  - $\geq$ or $\leq$ means to include that value.
  - $>$ or $<$ means to exclude that value.
- You can represent each condition by a section of the graph with open boundary points ($>$ or $<$) or closed boundary points ($\geq$ or $\leq$).

Graphing your piecewise-defined function might also help in making sense of the situation.

Try This

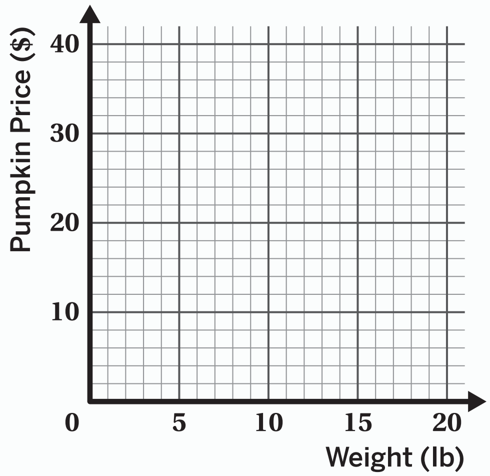
Here are the prices for pumpkins at a local farm:

- Pumpkins less than 5 pounds: $10
- Pumpkins greater than or equal to 15 pounds: $30
- All other pumpkins: $20

a Complete the piecewise-defined function to represent the price of pumpkins at this farm.

$$f(x) = \begin{cases} 10 & \square \\ \square & 5 \leq x < 15 \\ 30 & \square \end{cases}$$

b Sketch a graph of the function.



Summary | Lesson 13

There are several ways we can define, or describe, a *sequence*. When you define a sequence recursively, you are determining each term using the previous term.

You can define a sequence recursively by identifying the first term of the sequence and writing a rule for how the sequence changes between terms by either a *constant ratio* or a *constant difference.* When writing the rule in function notation, you can write the recursive definition by referencing the previous term, which can be written as $f(n-1)$.

Here is an example of a recursive definition for the sequence $32, 16, 8, 4, 2, 1$ written in function notation.

$$f(n) = \begin{cases} 32 & n = 1 \\ 0.5\, f(n-1) & n \geq 2 \end{cases}$$

The first term is 32, and the sequence changes by a constant ratio of 0.5. The rule is to multiply the previous term $f(n-1)$ by 0.5.

The expression $32(0.5)^{n-1}$ could also be used to define this sequence.

Try This

Determine which recursive definition matches this sequence:

$2, 5, 8, 11, 14, 17, 20, 23$

A. $f(1) = 2$

$f(n) = 3f(n-1)$

B. $f(1) = 23$

$f(n) = f(n-1) + 3$

C. $f(1) = 2$

$f(n) = f(n-1) + 3$

D. $f(1) = 5$

$f(n) = 2f(n-1)$

Summary | Lesson 14

You can define a sequence explicitly or recursively using function notation. To define a sequence recursively as a function, you need to know the value when the input is 1 and how the function changes from one term to another (for example, with a constant ratio of 3).

Here are two sequences defined recursively and explicitly.

Sequence	Recursive Definition	Explicit Definition
15, 12, 9, 6, 3, . . .	$a(1) = 15$ $a(n) = a(n-1) - 3$	$a(n) = 18 - 3n$ or $a(n) = 15 - 3(n-1)$
<table><tr><th>Term, n</th><th>$b(n)$</th></tr><tr><td>1</td><td>20</td></tr><tr><td>2</td><td>10</td></tr><tr><td>3</td><td>5</td></tr><tr><td>4</td><td>2.5</td></tr></table>	$b(1) = 20$ $b(n) = \frac{1}{2} \cdot b(n-1)$	$b(n) = 40 \cdot \left(\frac{1}{2}\right)^n$ or $b(n) = 20 \cdot \left(\frac{1}{2}\right)^{n-1}$

Try This

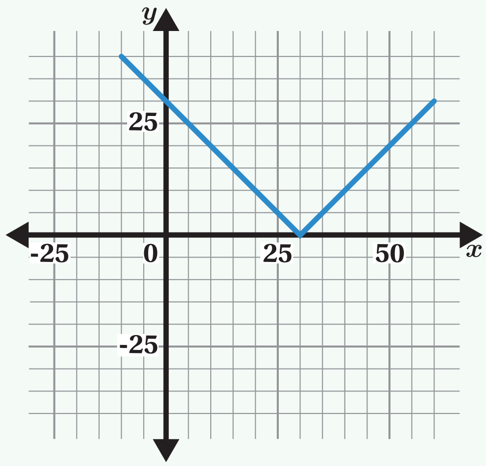
Write a recursive definition and an explicit definition for the sequence $f(n)$.

Sequence	Recursive Definition	Explicit Definition
<table><tr><th>Term, n</th><th>$f(n)$</th></tr><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>5</td></tr><tr><td>3</td><td>8</td></tr><tr><td>4</td><td>11</td></tr></table>	$f(1) =$ __________ $f(n) = f(n-1)$ __________	$f(n) =$ __________

Summary | Lesson 15

The output of an **absolute value function** is the distance of its input from a given value. The equation of an absolute value function is defined using absolute value symbols, and its graph forms the shape of a V. We can write absolute value functions in the form $f(x) = |x - h|$, where $f(x)$ gives the distance of any input, x, from h. Let's look at an example.

Mr. DeAndre asked his students to guess a mystery number and gave each student a score. Their score was how far away their guess was from his mystery number, 30.



Here is the graph of the function $f(x) = |x - 30|$, which gives the score for each guess, x.

We can use the equation to determine the value of $f(25)$ and interpret its meaning.

$$f(25) = |25 - 30|$$
$$= |-5|$$
$$= 5$$

This means a student who guessed 25 was 5 away from the mystery number.

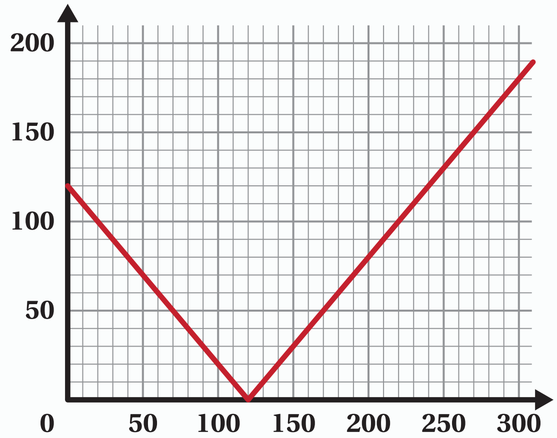
Try This

A carnival offers a prize for guessing the correct number of candies in a jar.

The function $a(x) = |x - 119|$ represents a person's score for a guess of x candies.

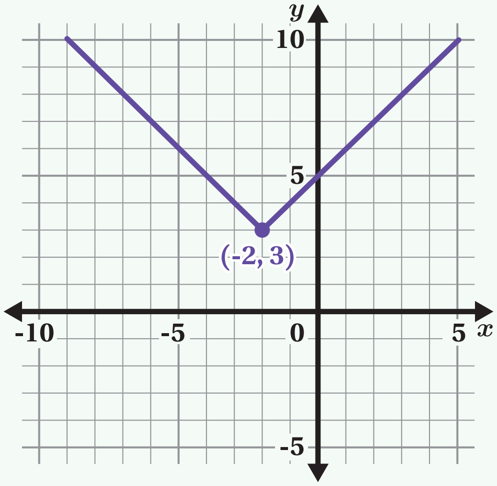
a What is the value of $a(120)$?

b What does $a(120)$ mean in this situation?



You can determine key features of the graph of an *absolute value function* by analyzing its table or equation, which are both helpful in sketching its graph.

Here are the graph and table for the absolute value function $f(x) = |x + 2| + 3$.



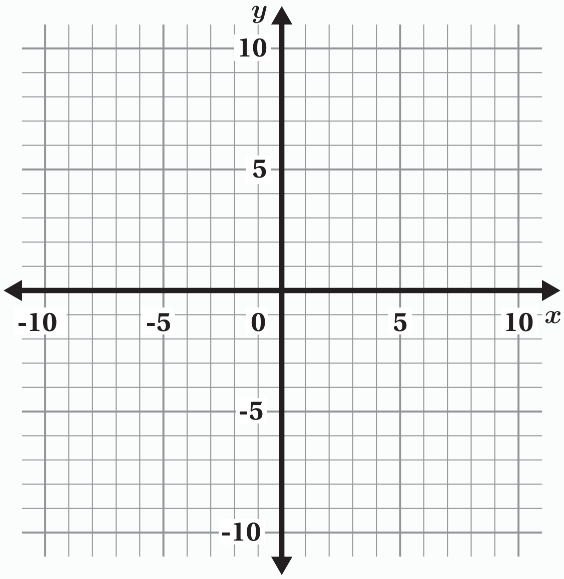
x	$f(x) = \|x + 2\| + 3$
-4	5
-2	3
0	5
2	7

Evaluating $f(x)$ at x = -2 makes the equation equal to 3. This means that when the input of the function is -2, the output is 3 and a point on the graph of $f(x)$ is (-2, 3). The values in the table show that there is symmetry around the point (-2, 3). This tells us that (-2, 3) is the minimum value of the function, which we can see by looking at the graph.

Try This

a Make a graph of the function $a(x) = |x + 3| + 1$. Create a table if it helps with your thinking.

x	$a(x) = \|x + 3\| + 1$



b What is the minimum value of this function? Explain your thinking.

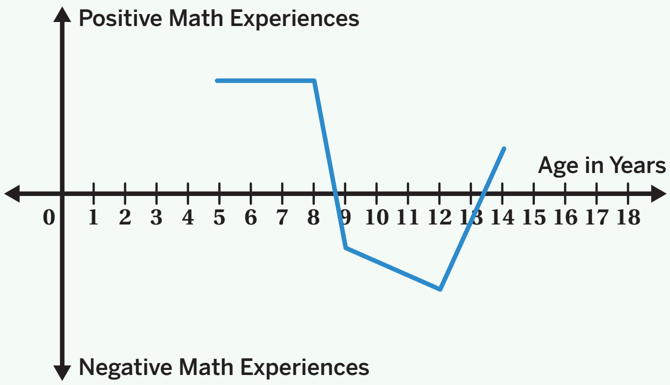
Summary | Lesson 17

Storytelling is a powerful way to learn more about others and to reflect on your own journey. Graphing a story can help us see interesting self-discoveries and have deeper discussions.

When you use equations, tables, words, graphs, and their key features to represent real-world relationships, pay close attention to the scale and units. You can look for the maximums or minimums; intercepts; intervals where the graph increases, decreases, or remains constant; and domain and range to make sense of the situation or someone's story. Let's look at an example.

Here's a graph of someone's math experiences over time. From the graph, we can learn that:

- Most of their positive math experiences were from ages 5 to 8, and their most negative math experience was at age 12.
- Their math experiences decreased from age 8 to 12 and increased after age 12.
- This person graphed their experiences from age 5 to age 14. It's possible they drew this graph at age 14.

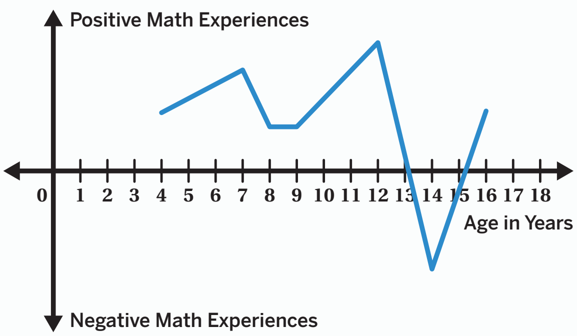


There is also a lot we can't tell from the graph of someone's story. For example, we don't know what the positive or negative math experiences were, or what emotions they were feeling at the time. The graph gives us only a window into someone else's story, not the full image.

Try This

Joel is an 11th grade student. He graphed his math experiences, $f(x)$, as a function of age, x.

Write a story about Joel's math experiences using some of these terms:



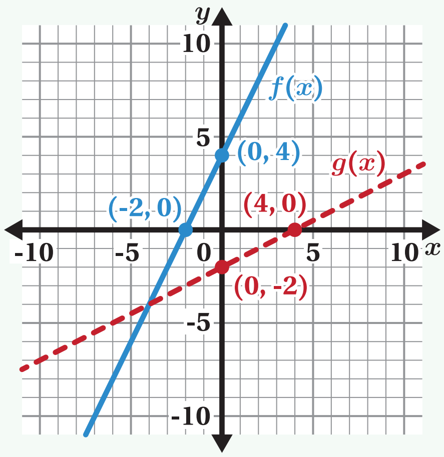
Word Bank		
increase	decrease	minimum
domain	range	maximum

Two functions are **inverses** of each other if their input-output pairs are reversed, and the graph of one function can be reflected onto the other over the line $y = x$. In general, if a function has a point at (h, k), then the inverse function has a point at (k, h).

Here is a linear function $f(x) = 2x + 4$.

One strategy you can use to determine the inverse of $f(x)$ is:

1. Choose two points on $f(x)$ and switch the x- and y-values of each point.
2. Then draw a line that goes through those points, and determine the slope and y-intercept.



Points on $f(x)$	Points on $g(x)$	Calculate the slope and y-intercept.	Write the equation of the inverse, $g(x)$.
(-2, 0)	(0, -2)	$\frac{-2-0}{0-4} = \frac{1}{2}$	The inverse of $f(x)$ is $g(x) = \frac{1}{2}x - 2$.
(0, 4)	(4, 0)	y-intercept: (0, -2)	

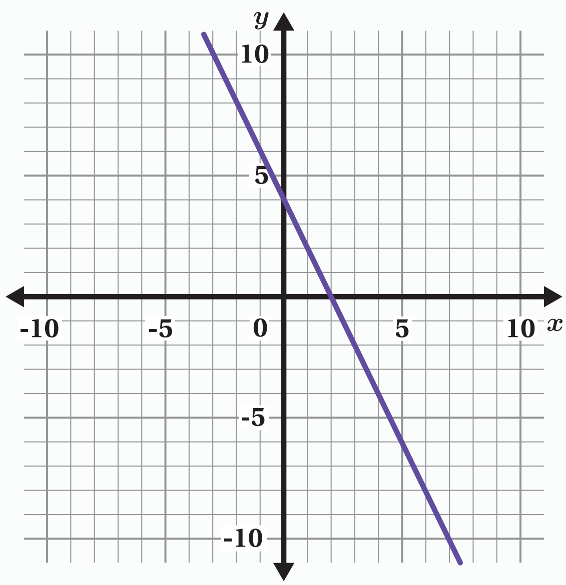
Try This

Here is the graph of $f(x) = -2x + 4$.

Use the graph to write an equation for the inverse function.

Draw a graph of the inverse function if it helps your thinking.

$g(x) =$ ____________________



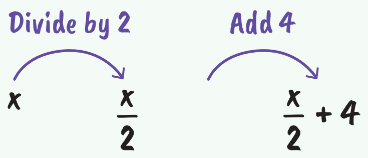
Summary | Lesson 19

One strategy you can use to determine the equation of an inverse function is to determine the operations of the original function, and then apply the inverse operations in reverse order.

For example, here is the function $f(x) = \frac{x}{2} + 4$. $f(x)$ and $g(x)$ are inverses.

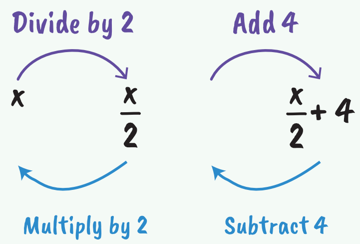
To find $g(x)$, first you can determine the operations acting on $f(x)$.

$$f(x) = \frac{x}{2} + 4$$



Then you can determine the inverse operations in reverse order.

$$f(x) = \frac{x}{2} + 4$$



The inverse function $g(x)$ will first subtract by 4, then multiply by 2. The inverse of $f(x)$ is $g(x) = 2(x - 4)$.

Try This

$h(x) = 3x + 9$.

$g(x)$ is the inverse of $h(x)$.

Determine the equation for $g(x)$.

$g(x) =$ ____________________

Lesson 1

No. *Explanations vary*. The input of 2 has multiple possible outputs (4 and 1), which means this is not a function.

Lesson 2

a. $w(2) = 8$
Caregiver Note: One strategy is to substitute 2 for x in the equation.
$w(2) = 2.25(2) + 3.5$
$w(2) = 4.5 + 3.5$
$w(2) = 8$

b. *Responses vary*. A waffle cone with 4 scoops of ice cream costs $\$12.50$.

Lesson 3

a. $b(x) = 1.75 + 2.25x$

b. $\$8.50$

Caregiver Note: One strategy is to substitute 3 for x in the equation
$b(x) = 1.75 + 2.25x$.

$b(3) = 1.75 + 2.25(3)$

$b(3) = 1.75 + 6.75$

$b(3) = 8.5$

Lesson 4

a. Jasmine's distance from home is the same at 15 minutes as at 20 minutes because she is in the library.

b. $d(50)$ represents the distance Jasmine was from home after 50 minutes, which was 2 miles.

Lesson 5

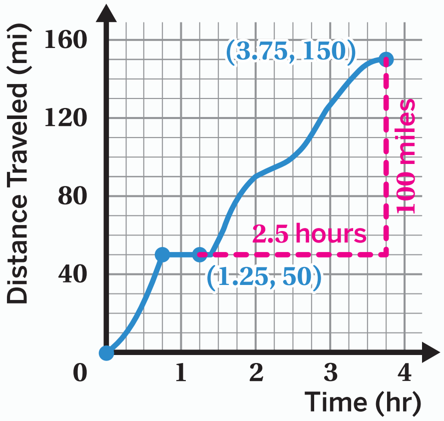
A. Is positive when $x > -2$.

C. Is decreasing when $x > 2$.

D. Has a minimum at $(-4, -4)$.

Lesson 6

40 miles per hour. *Explanations vary*. Here is one strategy for determining the average rate of change:



$\frac{100}{2.5} = 40$

Lesson 7

B. $f(10) > g(10)$

C. $f(17) = g(17)$

D. $g(5) = 1$

Lesson 8

a C. 1

D. 4

b *Responses vary*. Whole numbers greater than or equal to 1.

Lesson 9

Domain: $-3 \le x \le 4$

Range: $-2 \le f(x) \le 5$

Lesson 10

a $-3 \leq x \leq 6$

b $1 \leq h(x) \leq 7$

Lesson 11

a *Responses vary.* Different rules apply to different intervals of the function's domain. When x is between 0 and 3, $a(x) = x$. But when x is 3 or above, $a(x) = 2$.

b 1

Caregiver Note: One strategy is to look at the graph where $x = 1$. At $x = 1$, the graph passes through the point $(1, 1)$, so $a(1) = 1$.

c 2

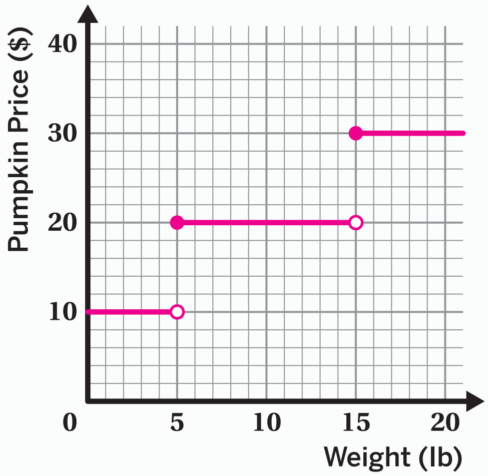
Caregiver Note: One strategy is to use the equation. For values of x greater than or equal to 3, the function is equal to 2.

Lesson 12

a

$$f(x) = \begin{cases} 10 & x < 5 \\ 20 & 5 \leq x < 15 \\ 30 & x \geq 15 \end{cases}$$

b



Lesson 13

C. $f(1) = 2$

$f(n) = f(n-1) + 3$

Lesson 14

Sequence	Recursive Definition	Explicit Definition
<table><tr><th>Term, n</th><th>$f(n)$</th></tr><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>5</td></tr><tr><td>3</td><td>8</td></tr><tr><td>4</td><td>11</td></tr></table>	$f(1) = \mathbf{2}$ $f(n) = f(n-1)\ \mathbf{+ 3}$	$f(n) = \mathbf{3n - 1}$

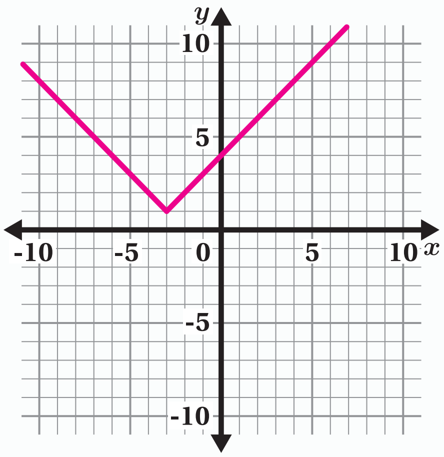
Lesson 15

a $a(120) = 1$

b *Responses vary*. If someone guessed 120, they were 1 away from the correct number of candies in the jar.

Lesson 16

a



b (-3, 1). *Explanations vary*. The minimum value is (-3, 1) because substituting -3 for x into $a(x)$ makes the absolute value expression equal to 0. When the input is -3, the output is 1 which tells us (-3, 1) is the minimum of the graph.

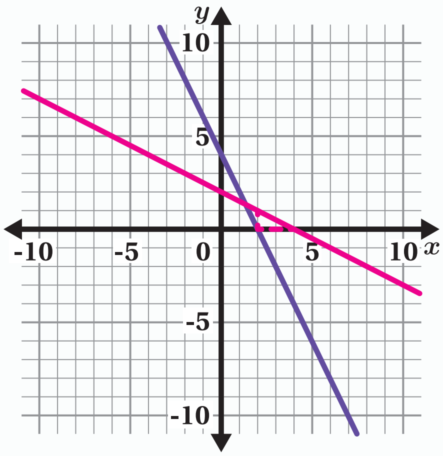
Lesson 17

Responses vary. Joel had positive math experiences from ages 4 to 13, with a maximum at age 12 (which is when he had his best math experience). His positive math experiences increased from ages 4 to 7 and 9 to 12. They decreased from ages 12 to 14, with a minimum at age 14 (which is when he had his worst math experience). The domain of the graph is from 4 to 16, which represents all the ages that Joel had math class so far.

Lesson 18

$g(x) = -\frac{1}{2}x + 2$

Caregiver Note: Here is one strategy: Two points on $f(x)$ are $(1, 2)$ and $(0, 4)$. Switching the x- and y- values gives two points on $g(x)$: $(2, 1)$ and $(4, 0)$. We can draw a line through those points on the graph:



To determine the slope, we can calculate the change in y-coordinates divided by the change in x-coordinates: $\frac{(1-0)}{(2-4)} = \frac{1}{-2} = -\frac{1}{2}$.

We can then use the graph to see that the y-intercept is at $(0, 2)$. This means that the equation of the line is $g(x) = -\frac{1}{2}x + 2$.

Lesson 19

$g(x) = \frac{(x-9)}{3}$

Caregiver Note: The operations of the original function are to first multiply by 3, then add 9. Applying the inverse operations in reverse order results in first subtracting 9, then dividing by 3.